

**تمرين 1:** احسب النهايات التالية:

$$\lim_{x \rightarrow +\infty} \sqrt{\sqrt{x}+1} - \sqrt{x+\sqrt{x}}$$

$$\lim_{x \rightarrow -\infty} x+3 - \sqrt{x^2+4x}$$

$$\lim_{x \rightarrow +\infty} x+3 - \sqrt{x+3}$$

$$m \in \mathbb{R} \text{ حيث } \lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1} - mx)$$

$$\lim_{x \rightarrow +\infty} \sqrt{1+2x^3} - \sqrt{x^3+x+1}$$

$$\lim_{x \rightarrow +\infty} \frac{\cos(x) + 5 \sin(x^2)}{x^4 + x^2 + 1}$$

$$\lim_{x \rightarrow -\infty} \left( \frac{x+1 - \sqrt{1-x}}{x^2 - \sqrt{x^2+2}} \right)$$

$$\lim_{x \rightarrow +\infty} \left( \frac{x - \sqrt{x^2+1}}{x^2 - \sqrt{x^4+1}} \right)$$

**تمرين 2:** احسب النهايات التالية:

$$\lim_{x \rightarrow -2^-} \left( \frac{\sqrt{x^2-4}}{x+2} \right)$$

$$\lim_{x \rightarrow 2^+} \left( \frac{\sqrt{x^2-4}}{x-2} \right)$$

$$\lim_{x \rightarrow -1} \left( \frac{\sqrt{1-3x-2}}{x+1} \right)$$

$$\lim_{x \rightarrow 0^+} \left( \frac{1}{x} + \cos\left(\frac{1}{x}\right) \right)$$

$$\lim_{x \rightarrow 1} \left( \sin\left(\frac{5}{1-x^3}\right) (x^2 - 2x + 1) \right)$$

$$\lim_{x \rightarrow 1} \left( \frac{x^2 + x - 2}{x^3 + 4x^2 - 8x + 3} \right)$$

**تمرين 3:** احسب النهايات التالية:

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{3} \cos x - \sin^2 x - \sqrt{3}}{x^2} \right)$$

$$\lim_{x \rightarrow 3} \frac{\sin(\pi x)}{x-3}$$

$$\lim_{x \rightarrow 0} \left( \frac{1}{\sqrt{3} \cos x - \sin^2 x - \sqrt{3}} \right)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sqrt{1-\cos(x)} - \sqrt{1-\sin(x)}}{1-\tan(x)} \right)$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\tan x - 1}{2 \cos x - \sqrt{2}} \right)$$

$$\lim_{x \rightarrow \frac{\pi}{6}} \left( \frac{\cos x - \sqrt{3} \sin x}{6x - \pi} \right)$$

**تمرين 4:** ليكن  $a \in \mathbb{R}$  و  $f_a$  الدالة العددية المعرفة بما يلي:

$$f_a(x) = \frac{1}{x+a} - \frac{a^2 x^2}{x^3 + a^3}$$

1) حدد مجموعة تعريف الدالة  $f_a$ 2) حدد قيم العدد  $a$  التي من أجلها تقبل  $f_a$  نهاية في  $-a$ **تمرين 5:** ليكن  $m$  عددا حقيقيا ولتكن  $f$  الدالة العددية المعرفة بما يلي:

$$f(x) = \frac{mx^3 + (m-2)x^2 + (m-1)x + m-3}{x(x-2)(x-3)}$$

1) حدد مجموعة تعريف الدالة  $f$ 2) ادرس نهاية الدالة  $f$  عند  $+\infty$  و  $-\infty$  و  $0$  حسب قيم البارامتر  $m$ **تمرين 6:** لتكن  $f$  الدالة العددية المعرفة بما يلي:

$$f(x) = \frac{x^2 \sqrt{x+2} - 8}{4-x^2}$$

1) حدد مجموعة تعريف الدالة  $f$ 2) احسب  $\lim_{x \rightarrow +\infty} f(x)$ 3) بين أن  $f$  تقبل تمديدا بالاتصال في  $2$  وحدده.

## تمرين 1 :

$$\lim_{x \rightarrow +\infty} x+3-\sqrt{x+3} = \lim_{x \rightarrow +\infty} (\sqrt{x+3})^2 - \sqrt{x+3} = \lim_{x \rightarrow +\infty} \sqrt{x+3} (\sqrt{x+3} - 1) = +\infty$$

$$\begin{aligned} \lim_{x \rightarrow +\infty} x+3-\sqrt{x^2+4x} &= \lim_{x \rightarrow +\infty} x+2-\sqrt{x^2+4x}+1 = \lim_{x \rightarrow +\infty} \frac{x^2+4x+4-x^2-4x}{x+2+\sqrt{x^2+4x}}+1 \\ &= \lim_{x \rightarrow +\infty} \frac{4}{x+2+\sqrt{x^2+4x}}+1 = 0+1=1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow +\infty} \sqrt{\sqrt{x}+1}-\sqrt{x+\sqrt{x}} &= \lim_{x \rightarrow +\infty} \sqrt{\sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)}-\sqrt{x\left(1+\frac{\sqrt{x}}{x}\right)} \\ &= \lim_{x \rightarrow +\infty} \sqrt{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}-\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}} = \lim_{x \rightarrow +\infty} \sqrt{x}\left(\frac{\sqrt{\sqrt{x}}}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}-\sqrt{1+\frac{1}{\sqrt{x}}}\right) \\ &= \lim_{x \rightarrow +\infty} \sqrt{x}\left(\frac{1}{\sqrt{\sqrt{x}}}\sqrt{1+\frac{1}{\sqrt{x}}}-\sqrt{1+\frac{1}{\sqrt{x}}}\right) = -\infty \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow +\infty} \sqrt{1+2x^3}-\sqrt{x^3+x+1} &= \lim_{x \rightarrow +\infty} \sqrt{x^3}\sqrt{\frac{1}{x^3}+2}-\sqrt{x^3}\sqrt{1+\frac{1}{x^2}+\frac{1}{x^3}} \\ &= \lim_{x \rightarrow +\infty} \sqrt{x^3}\left(\sqrt{\frac{1}{x^3}+2}-\sqrt{1+\frac{1}{x^2}+\frac{1}{x^3}}\right) = +\infty \end{aligned}$$

▪ إذا كان  $m > 0$  فإن :  $\lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1}-mx) = +\infty$  (  $\lim_{x \rightarrow -\infty} -mx = +\infty$  )

▪ إذا كان  $m = 0$  فإن :  $\lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1}-mx) = \lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1}) = +\infty$

▪ إذا كان  $-\sqrt{5} < m < 0$  فإن :

$$\begin{aligned} \lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1}-mx) &= \lim_{x \rightarrow -\infty} \left( \sqrt{x^2\left(5+\frac{1}{x}+\frac{1}{x^2}\right)}-mx \right) = \lim_{x \rightarrow -\infty} \left( -x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-mx \right) \\ &= \lim_{x \rightarrow -\infty} -x\left(\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}+m\right) = +\infty \end{aligned}$$

▪ إذا كان  $m = -\sqrt{5}$  فإن :

$$\begin{aligned} \lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1}-mx) &= \lim_{x \rightarrow -\infty} \sqrt{5x^2+x+1}+\sqrt{5}x = \lim_{x \rightarrow -\infty} \frac{x+1}{\sqrt{5x^2+x+1}-\sqrt{5}x} \\ &= \lim_{x \rightarrow -\infty} \frac{x\left(1+\frac{1}{x}\right)}{-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{5}x} = \lim_{x \rightarrow -\infty} \frac{1+\frac{1}{x}}{-x\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}-\sqrt{5}x} = \frac{-1}{2\sqrt{5}} \end{aligned}$$

▪ إذا كان  $m < -\sqrt{5}$  فإن :  $\lim_{x \rightarrow -\infty} (\sqrt{5x^2+x+1}-mx) = \lim_{x \rightarrow -\infty} -x\left(\sqrt{5+\frac{1}{x}+\frac{1}{x^2}}+m\right) = -\infty$

$$\lim_{x \rightarrow +\infty} \left( \frac{x - \sqrt{x^2 + 1}}{x^2 - \sqrt{x^4 - 1}} \right) = \lim_{x \rightarrow +\infty} \frac{(x^2 - (x^2 + 1))(x^2 + \sqrt{x^4 - 1})}{(x^4 - (x^4 - 1))(x + \sqrt{x^2 + 1})} = \lim_{x \rightarrow +\infty} \frac{-\left(x^2 + x^2 \sqrt{1 - \frac{1}{x^4}}\right)}{\left(x + x \sqrt{1 + \frac{1}{x^2}}\right)}$$

$$\lim_{x \rightarrow +\infty} \left( \frac{x - \sqrt{x^2 + 1}}{x^2 - \sqrt{x^4 - 1}} \right) = \lim_{x \rightarrow +\infty} \frac{-x \left(1 + \sqrt{1 - \frac{1}{x^4}}\right)}{1 + \sqrt{1 + \frac{1}{x^2}}} = -\infty$$

$$\lim_{x \rightarrow -\infty} \left( \frac{x + 1 - \sqrt{1 - x}}{x^2 - \sqrt{x^2 + 2}} \right) = \lim_{x \rightarrow -\infty} \frac{x \left(1 + \frac{1}{x} - \frac{\sqrt{1 - x}}{x}\right)}{x^2 + x \sqrt{1 + \frac{2}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{1 + \frac{1}{x} + \sqrt{\frac{1 - x}{x^2}}}{x \left(1 + \frac{1}{x} \sqrt{1 + \frac{2}{x^2}}\right)} = 0$$

$$\forall x \in \mathbb{R} \quad |\cos x + 5 \sin x^2| \leq |\cos x| + |5 \sin x^2| \leq 1 + 5 \leq 6 \quad \text{لدينا :}$$

$$\text{منه :} \quad \forall x \in \mathbb{R} \quad \left| \frac{\cos x + 5 \sin x^2}{x^4 + x^2 + 1} \right| \leq \frac{6}{x^4 + x^2 + 1} \quad \text{وبما أن} \quad \lim_{x \rightarrow +\infty} \frac{1}{x^4 + x^2 + 1} = 0 \quad \text{فإن :}$$

$$\lim_{x \rightarrow +\infty} \frac{\cos x + 5 \sin x^2}{x^4 + x^2 + 1} = 0$$

**تمرين 2 :**

$$\lim_{x \rightarrow -1} \left( \frac{\sqrt{1 - 3x} - 2}{x + 1} \right) = \lim_{x \rightarrow -1} \frac{1 - 3x - 4}{(x + 1)(\sqrt{1 - 3x} + 2)} = \lim_{x \rightarrow -1} \frac{-3}{\sqrt{1 - 3x} + 2} = \frac{-3}{4}$$

$$\lim_{x \rightarrow 2^+} \frac{\sqrt{x^2 - 4}}{x - 2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{(x - 2)(x + 2)}}{(x - 2)^2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{x + 2}}{x - 2} = +\infty$$

$$\lim_{x \rightarrow 2^-} \frac{\sqrt{x^2 - 4}}{x + 2} = \lim_{x \rightarrow 2^-} \frac{\sqrt{x^2 - 4}}{-(x + 2)} = \lim_{x \rightarrow 2^+} \frac{\sqrt{(x - 2)(x + 2)}}{(x + 2)^2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{x - 2}}{x + 2} = -\infty$$

$$\lim_{x \rightarrow 1} \left( \frac{x^2 + x - 2}{x^3 + 4x^2 - 8x + 3} \right) = \lim_{x \rightarrow 1} \frac{(x - 1)(x + 2)}{(x - 1)(x^2 + 5x - 3)} = 1$$

$$\lim_{x \rightarrow 1} \left( \sin \left( \frac{5}{1 - x^3} \right) (x^2 - 2x + 1) \right) = \lim_{x \rightarrow 1} \sin \left( \frac{5}{1 - x^3} \right) (x - 1)^2 = 0$$

$$\forall x \in \mathbb{R} \quad \left| \sin \left( \frac{5}{1 - x^3} \right) \right| \leq 1 \Rightarrow \forall x \in \mathbb{R} \quad \left| \sin \left( \frac{5}{1 - x^3} \right) (x - 1)^2 \right| \leq (x - 1)^2 \quad \text{لأن :}$$

$$\text{و} \quad \lim_{x \rightarrow 1} (x - 1)^2 = 0 \quad \text{وذلك حسب مصاديق تقارب نهاية دالة.}$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} + 1 = +\infty \quad \text{و} \quad \forall x > 0 \quad \cos \left( \frac{1}{x} \right) \geq -1 \Rightarrow \forall x > 0 \quad \frac{1}{x} + \cos \left( \frac{1}{x} \right) \geq \frac{1}{x} - 1 \quad \text{لأن :} \quad \lim_{x \rightarrow 0^+} \frac{1}{x} + \cos \left( \frac{1}{x} \right) = +\infty$$

وذلك حسب مصاديق تقارب نهاية دالة.

**تمرين 3 :**



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رفيقكم الدائم

$$\begin{aligned} \lim_{x \rightarrow 0} \left( \frac{1}{\sqrt{3} \cos x - \sin^2 x - \sqrt{3}} \right) &= \lim_{x \rightarrow 0} \left( \frac{1}{\sqrt{3}(\cos x - 1) - \sin^2 x} \right) = \lim_{x \rightarrow 0} \left( \frac{-1}{\sqrt{3}(1 - \cos(x)) + \sin^2 x} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{-\frac{1}{x^2}}{\sqrt{3} \left( \frac{1 - \cos(x)}{x^2} \right) + \left( \frac{\sin x}{x} \right)^2} \right) = -\infty \\ & \left( \lim_{x \rightarrow 0} -\frac{1}{x^2} = -\infty \text{ و } \lim_{x \rightarrow 0} \sqrt{3} \left( \frac{1 - \cos(x)}{x^2} \right) + \left( \frac{\sin x}{x} \right)^2 = \frac{\sqrt{3}}{2} + 1 \text{ لأن: } \right) \end{aligned}$$

$$\lim_{x \rightarrow 0} \left( \frac{\sqrt{3} \cos x - \sin^2 x - \sqrt{3}}{x^2} \right) = \lim_{x \rightarrow 0} \left( -\sqrt{3} \left( \frac{1 - \cos(x)}{x^2} \right) - \left( \frac{\sin(x)}{x} \right)^2 \right) = \frac{-\sqrt{3}}{2} - 1$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{6}} \left( \frac{\cos x - \sqrt{3} \sin x}{6x - \pi} \right) &= \lim_{x \rightarrow \frac{\pi}{6}} \left( \frac{2 \left( \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \right)}{6 \left( x - \frac{\pi}{6} \right)} \right) = \lim_{x \rightarrow \frac{\pi}{6}} \left( \frac{1 \left( \sin \left( \frac{\pi}{6} \right) \cos x - \cos \left( \frac{\pi}{6} \right) \sin x \right)}{3 \left( x - \frac{\pi}{6} \right)} \right) \\ &= \lim_{x \rightarrow \frac{\pi}{6}} \left( \frac{1 \sin \left( \frac{\pi}{6} - x \right)}{3 \left( x - \frac{\pi}{6} \right)} \right) = \lim_{x \rightarrow \frac{\pi}{6}} \left( \frac{-1 \sin \left( x - \frac{\pi}{6} \right)}{3 \left( x - \frac{\pi}{6} \right)} \right) = \lim_{t \rightarrow 0} \left( \frac{-1 \sin(t)}{3 t} \right) = \frac{-1}{3} \end{aligned}$$

قمنا بتغيير المتغير  $x$  وذلك بوضع  $t = x - \frac{\pi}{6}$ ، كما يمكن إجراء تغيير المتغير منذ البداية. 🌱

$$\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\tan x - 1}{2 \cos x - \sqrt{2}} \right) = \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\tan x - \tan \left( \frac{\pi}{4} \right)}{2 \cos x - \sqrt{2}} \right) = \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\left( 1 + \tan(x) \tan \left( \frac{\pi}{4} \right) \right) \tan \left( x - \frac{\pi}{4} \right)}{2 \cos x - \sqrt{2}} \right) \quad \text{نضع } t = x - \frac{\pi}{4}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\tan x - 1}{2 \cos x - \sqrt{2}} \right) &= \lim_{t \rightarrow 0} \left( \frac{\left( 1 + \tan \left( t + \frac{\pi}{4} \right) \right) \tan(t)}{2 \cos \left( t + \frac{\pi}{4} \right) - \sqrt{2}} \right) = \lim_{t \rightarrow 0} \left( \frac{\left( 1 + \tan \left( t + \frac{\pi}{4} \right) \right) \tan(t)}{2 \left( \cos(t) \frac{\sqrt{2}}{2} - \sin(t) \frac{\sqrt{2}}{2} \right) - \sqrt{2}} \right) \\ &= \lim_{t \rightarrow 0} \left( \frac{\left( 1 + \tan \left( t + \frac{\pi}{4} \right) \right) \tan(t)}{-\sqrt{2}(1 - \cos(t)) - \sqrt{2} \sin(t)} \right) = \lim_{t \rightarrow 0} \left( \frac{\left( 1 + \tan \left( t + \frac{\pi}{4} \right) \right) \frac{\tan(t)}{t}}{-\sqrt{2} t \frac{(1 - \cos(t))}{t^2} - \sqrt{2} \frac{\sin(t)}{t}} \right) \\ &= \frac{(1+1) \times 1}{-\sqrt{2} \times 0 \times \frac{1}{2} - \sqrt{2} \times 1} = -\sqrt{2} \end{aligned}$$

لاحظ أن استعمال الخاصية  $\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$  🌱

بالشكل  $\tan(a) - \tan(b) = (1 + \tan(a)\tan(b))\tan(a - b)$  أفضل من استعمال تغيير المتغير من البداية. 🌱



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$$\begin{aligned}
\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sqrt{1 - \cos(x)} - \sqrt{1 - \sin(x)}}{1 - \tan(x)} \right) &= \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sin(x) - \cos(x)}{(1 + \tan(x)) \tan\left(x - \frac{\pi}{4}\right)} \times \frac{1}{\sqrt{1 - \cos(x)} + \sqrt{1 - \sin(x)}} \right) \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sqrt{2} \left( \sin(x) \cos\left(\frac{\pi}{4}\right) - \cos(x) \sin\left(\frac{\pi}{4}\right) \right)}{(1 + \tan(x)) \tan\left(x - \frac{\pi}{4}\right)} \times \frac{1}{\sqrt{1 - \cos(x)} + \sqrt{1 - \sin(x)}} \right) \\
&= \lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sin\left(x - \frac{\pi}{4}\right)}{\tan\left(x - \frac{\pi}{4}\right)} \times \frac{\sqrt{2}}{(1 + \tan(x)) (\sqrt{1 - \cos(x)} + \sqrt{1 - \sin(x)})} \right) \\
\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x - \frac{\pi}{4}\right)}{\tan\left(x - \frac{\pi}{4}\right)} &= \lim_{t \rightarrow 0} \frac{\sin(t)}{\tan(t)} = \lim_{t \rightarrow 0} \frac{\sin(t)}{t} = \frac{1}{1} = 1 \quad \text{وبما أن:}
\end{aligned}$$

$$\begin{aligned}
\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sqrt{2}}{(1 + \tan(x)) (\sqrt{1 - \cos(x)} + \sqrt{1 - \sin(x)})} \right) &= \frac{\sqrt{2}}{(1+1) \left( \sqrt{1 - \frac{\sqrt{2}}{2}} + \sqrt{1 - \frac{\sqrt{2}}{2}} \right)} \\
&= \frac{\sqrt{2}}{4 \sqrt{\frac{2 - \sqrt{2}}{2}}} = \frac{\sqrt{4 + 2\sqrt{2}}}{4}
\end{aligned}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sqrt{1 - \cos(x)} - \sqrt{1 - \sin(x)}}{1 - \tan(x)} \right) = \frac{\sqrt{4 + 2\sqrt{2}}}{4} \quad \text{فإن:}$$

$$f_a(x) = \frac{1}{x+a} - \frac{a^2 x^2}{x^3 + a^3} : \text{تمرين 4}$$

$$Df_a = \mathbb{R}_{\{-a\}} \quad \text{منه} \quad x \in Df_a \Leftrightarrow \begin{cases} x+a \neq 0 \\ x^3 + a^3 \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq -a \\ x^3 \neq (-a)^3 \end{cases} \Leftrightarrow x \neq -a \quad (1) \text{ لدينا}$$

$$f_a(x) = \frac{1}{x+a} - \frac{a^2 x^2}{x^3 + a^3} = \frac{x^2 - ax + a^2}{x^3 + a^3} - \frac{a^2 x^2}{x^3 + a^3} = \frac{(1-a^2)x^2 - ax + a^2}{x^3 + a^3} : \text{لدينا} \quad (2)$$

$$g_a(-a) = (1-a^2)a^2 + a^2 + a^2 = 3a^2 - a^4 = a^2(3-a^2) : \text{اذن} \quad , \quad g_a(x) = (1-a^2)x^2 - ax + a^2 : \text{نضع}$$

$$\lim_{\substack{x \rightarrow 0 \\ x < 0}} f_0(x) = -\infty \quad \text{و} \quad \lim_{\substack{x \rightarrow 0 \\ x > 0}} f_0(x) = +\infty : \text{منه} \quad f_0(x) = \frac{x^2}{x^3} = \frac{1}{x} : \text{فان} \quad a = 0$$

$$g_a(-a) > 0 : \text{فان} \quad (a \in ]-\sqrt{3}; 0[ \cup ]0; \sqrt{3}[ \text{ أي} ) \quad a \neq 0 \quad \text{و} \quad a^2 < 3 : \text{إذا كان}$$

$$\lim_{\substack{x \rightarrow -a \\ x < -a}} f_a(x) = -\infty \quad \text{و} \quad \lim_{\substack{x \rightarrow -a \\ x > -a}} f_a(x) = +\infty : \text{منه}$$

$$g_a(-a) < 0 : \text{فان} \quad (a \in ]-\infty; -\sqrt{3}[ \cup ]\sqrt{3}; +\infty[ \text{ أي} ) \quad \text{و} \quad a^2 > 3 : \text{إذا كان}$$

$$\lim_{\substack{x \rightarrow -a \\ x < -a}} f_a(x) = +\infty \quad \text{و} \quad \lim_{\substack{x \rightarrow -a \\ x > -a}} f_a(x) = -\infty : \text{منه}$$

$$g_a(-\sqrt{3}) = 0 : \text{فان} \quad a = \sqrt{3} : \text{إذا كان} : \text{منه}$$

$$\lim_{x \rightarrow -\sqrt{3}} f_{\sqrt{3}}(x) = \lim_{x \rightarrow -\sqrt{3}} \frac{-2x^2 - \sqrt{3}x + 3}{x^3 + (\sqrt{3})^3} = \lim_{x \rightarrow -\sqrt{3}} \frac{(x+\sqrt{3})(\sqrt{3}-2x)}{(x+\sqrt{3})(x^2 - x\sqrt{3} + 3)} = \lim_{x \rightarrow -\sqrt{3}} \frac{\sqrt{3}-2x}{x^2 - x\sqrt{3} + 3} = \frac{3\sqrt{3}}{9} = \frac{\sqrt{3}}{3}$$

$$g_a(\sqrt{3}) = 0 : \text{فان} \quad a = -\sqrt{3} : \text{إذا كان} : \text{منه}$$

$$\lim_{x \rightarrow \sqrt{3}} f_{-\sqrt{3}}(x) = \lim_{x \rightarrow \sqrt{3}} \frac{-2x^2 + \sqrt{3}x + 3}{x^3 - (\sqrt{3})^3} = \lim_{x \rightarrow \sqrt{3}} \frac{(x-\sqrt{3})(-\sqrt{3}-2x)}{(x-\sqrt{3})(x^2 + x\sqrt{3} + 3)} = \lim_{x \rightarrow \sqrt{3}} \frac{-\sqrt{3}-2x}{x^2 + x\sqrt{3} + 3} = \frac{-3\sqrt{3}}{9} = \frac{-\sqrt{3}}{3}$$

خلاصة: القيم التي تجيب عن السؤال هي :  $a = \sqrt{3}$  و  $a = -\sqrt{3}$

استعملنا المحددة للتعميل

$$f(x) = \frac{mx^3 + (m-2)x^2 + (m-1)x + m-3}{x(x-2)(x-3)} \quad \text{تمرين 5:}$$

1) لدينا:  $x \in Df \Leftrightarrow x \neq 0 \text{ et } x-2 \neq 0 \text{ et } x-3 \neq 0$  ، إذن :  $Df = \mathbb{R}_{\{0;2;3\}}$

2) ندرس النهاية في اللانهاية:

▪ إذا كان  $m=0$  فإن:  $\lim_{x \rightarrow -\infty} f(x) = 0$  و  $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{-2x^2 - x - 3}{x(x-2)(x-3)} = \lim_{x \rightarrow +\infty} \frac{-2x^2}{x^3} = \lim_{x \rightarrow +\infty} \frac{-2}{x} = 0$

▪ إذا كان  $m \neq 0$  فإن:  $\lim_{x \rightarrow -\infty} f(x) = m$  و  $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{mx^2}{x^3} = m$

يمكن أن نلخص الحالات في النتيجة التالية،  $\forall m \in \mathbb{R} \quad \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = m$

نضع:  $g(x) = mx^3 + (m-2)x^2 + (m-1)x + m-3$

و  $g(0) = m-3$  و  $g(2) = 8m + 4(m-2) + 2(m-1) + m-3 = 15m-13$

و  $g(3) = 27m + 9(m-2) + 3(m-1) + m-3 = 40m-24$

ندرس النهاية في 0:

▪ إذا كان  $m=3$  فإن:  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{3x^3 + x^2 + 2x}{x(x-2)(x-3)} = \lim_{x \rightarrow 0} \frac{3x^2 + x + 2}{(x-2)(x-3)} = \frac{2}{6} = \frac{1}{3}$

▪ إذا كان  $m > 3$  فإن:  $\lim_{\substack{x \rightarrow 0 \\ x < 0}} f(x) = -\infty \left( \frac{\ell > 0}{0^-} \right)$  و  $\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x) = +\infty \left( \frac{\ell > 0}{0^+} \right)$

▪ إذا كان  $m < 3$  فإن:  $\lim_{\substack{x \rightarrow 0 \\ x < 0}} f(x) = +\infty \left( \frac{\ell < 0}{0^-} \right)$  و  $\lim_{\substack{x \rightarrow 0 \\ x > 0}} f(x) = -\infty \left( \frac{\ell < 0}{0^+} \right)$

$$f(x) = \frac{x^2\sqrt{x+2} - 8}{4 - x^2} \quad \text{تمرين 6:}$$

1) لدينا:  $x \in Df \Leftrightarrow \begin{cases} 4 - x^2 \neq 0 \\ x+2 \geq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq 2 \text{ et } x \neq -2 \\ x \geq -2 \end{cases} \Leftrightarrow \begin{cases} x \neq 2 \\ x > -2 \end{cases} \Leftrightarrow x \in ]-2; 2[ \cup ]2; +\infty[$

إذن :  $Df = ]-2; 2[ \cup ]2; +\infty[$

2) إذا كان  $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x^2\sqrt{x+2} - 8}{4 - x^2} = \lim_{x \rightarrow +\infty} \frac{x^2 \left( \sqrt{x+2} - \frac{8}{x^2} \right)}{x^2 \left( \frac{4}{x^2} - 1 \right)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x+2} - \frac{8}{x^2}}{\frac{4}{x^2} - 1} = -\infty$

3)  $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2\sqrt{x+2} - 8}{4 - x^2} = \lim_{x \rightarrow 2} \frac{x^2(\sqrt{x+2} - 2) + 2x^2 - 8}{4 - x^2} = \lim_{x \rightarrow 2} \frac{x^2 \frac{x+2-4}{\sqrt{x+2}+2} + 2(x^2 - 4)}{4 - x^2}$

$$= \lim_{x \rightarrow 2} \frac{x^2 \frac{x-2}{\sqrt{x+2}+2}}{4 - x^2} - 2 = \lim_{x \rightarrow 2} \frac{-x^2}{(x+2)(\sqrt{x+2}+2)} - 2 = \frac{-4}{16} - 2 = \frac{-9}{4}$$

إذن  $f$  تقبل تمديدا بالاتصال في 2 تمديده هو:

$$\begin{cases} f(x) = \frac{x^2\sqrt{x+2} - 8}{4 - x^2}; & x \in ]-2; 2[ \cup ]2; +\infty[ \\ f(2) = \frac{9}{4} \end{cases}$$